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# F.Sc Math Part 1 Solved Notes Unit 14

## Chapter 14: Solutions of Trigonometric Equations Solved Notes

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مزید نوٹس، گزشته پپر، ٹیسٹ پپر، گیس پپر، ذیت شیٹ، رزلٹ اور بہت سچے۔

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**Question # 1**

Find the solution set of the following equation which lies in  $[0, 2\pi]$

(i)  $\sin x = -\frac{\sqrt{3}}{2}$

(ii)  $\operatorname{cosec} \theta = 2$

(iv)  $\cot \theta = \frac{1}{\sqrt{3}}$

**Solution**

(i) Since  $\sin x = -\frac{\sqrt{3}}{2} \Rightarrow x = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$   
 $\Rightarrow x = \frac{5\pi}{3}, \frac{4\pi}{3}$  where  $x \in [0, 2\pi]$

(ii) Since  $\operatorname{cosec} \theta = 2$

$$\begin{aligned} \Rightarrow \frac{1}{\sin \theta} = 2 &\Rightarrow \sin \theta = \frac{1}{2} \\ \Rightarrow \theta = \sin^{-1}\left(\frac{1}{2}\right) &= \frac{\pi}{6}, \frac{5\pi}{6} \quad \text{where } \theta \in [0, 2\pi] \end{aligned}$$

(iii) *Do yourself*

(iv) Since  $\cot \theta = \frac{1}{\sqrt{3}}$   
 $\Rightarrow \frac{1}{\tan \theta} = \frac{1}{\sqrt{3}} \Rightarrow \tan \theta = \sqrt{3}$   
 $\Rightarrow \theta = \tan^{-1}(\sqrt{3})$   
 $\Rightarrow \theta = \frac{\pi}{3}, \frac{4\pi}{3}$  where  $\theta \in [0, 2\pi]$

**Question # 2**

Solve the following trigonometric equations:

(i)  $\tan^2 \theta = \frac{1}{3}$       (ii)  $\operatorname{cosec}^2 \theta = \frac{4}{3}$       (iii)  $\sec^2 \theta = \frac{4}{3}$       (iv)  $\cot^2 \theta = \frac{1}{3}$

**Solution**

(i) Since  $\tan^2 \theta = \frac{1}{3} \Rightarrow \tan \theta = \pm \frac{1}{\sqrt{3}}$

$$\Rightarrow \tan \theta = \frac{1}{\sqrt{3}} \quad \text{or} \quad \tan \theta = -\frac{1}{\sqrt{3}}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) \quad \text{or} \quad \theta = \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$$

$$\Leftrightarrow \theta = \frac{\pi}{6} \quad \text{or} \quad \theta = \frac{5\pi}{6}$$

Since period of  $\tan \theta$  is  $\pi$

Therefore general value of  $\theta = \frac{\pi}{6} + n\pi, \frac{5\pi}{6} + n\pi$

So Solution Set =  $\left\{\frac{\pi}{6} + n\pi\right\} \cup \left\{\frac{5\pi}{6} + n\pi\right\}$  where  $n \in \mathbb{Z}$

$$(ii) \text{ Since } \operatorname{cosec}^2 \theta = \frac{4}{3}$$

$$\Rightarrow \operatorname{cosec} \theta = \pm \frac{2}{\sqrt{3}}$$

$$\Rightarrow \operatorname{cosec} \theta = \frac{2}{\sqrt{3}} \quad \text{or} \quad \operatorname{cosec} \theta = -\frac{2}{\sqrt{3}}$$

$$\Rightarrow \sin \theta = \frac{\sqrt{3}}{2} \quad \text{or} \quad \sin \theta = -\frac{\sqrt{3}}{2}$$

$$\Rightarrow \theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) \quad \text{or} \quad \theta = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow \theta = \frac{\pi}{3}, \frac{2\pi}{3} \quad \text{or} \quad \theta = \frac{4\pi}{3}, \frac{5\pi}{3}$$

Since period of  $\operatorname{cosec} \theta$  is  $2\pi$

Therefore general value of  $\theta = \frac{\pi}{3} + 2n\pi, \frac{2\pi}{3} + 2n\pi, \frac{4\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi$

Solution set =  $\left\{\frac{\pi}{3} + 2n\pi\right\} \cup \left\{\frac{2\pi}{3} + 2n\pi\right\} \cup \left\{\frac{4\pi}{3} + 2n\pi\right\} \cup \left\{\frac{5\pi}{3} + 2n\pi\right\}$  where  $n \in \mathbb{Z}$ .

$$(iii) \text{ Since } \sec^2 \theta = \frac{4}{3}$$

$$\Rightarrow \sec \theta = \pm \frac{2}{\sqrt{3}}$$

$$\Rightarrow \sec \theta = \frac{2}{\sqrt{3}} \quad \text{or} \quad \sec \theta = -\frac{2}{\sqrt{3}}$$

$$\Rightarrow \cos \theta = \frac{\sqrt{3}}{2} \quad \text{or} \quad \cos \theta = -\frac{\sqrt{3}}{2}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) \quad \text{or} \quad \theta = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow \theta = \frac{\pi}{6}, \frac{11\pi}{6} \quad \text{or} \quad \theta = \frac{5\pi}{6}, \frac{7\pi}{6}$$

$\therefore$  period of  $\sec \theta$  is  $2\pi$

$\therefore$  general values of  $\theta = \frac{\pi}{6} + 2n\pi, \frac{5\pi}{6} + 2n\pi, \frac{7\pi}{6} + 2n\pi, \frac{11\pi}{6} + 2n\pi$

S. Set =  $\left\{\frac{\pi}{6} + 2n\pi\right\} \cup \left\{\frac{5\pi}{6} + 2n\pi\right\} \cup \left\{\frac{7\pi}{6} + 2n\pi\right\} \cup \left\{\frac{11\pi}{6} + 2n\pi\right\}$  where  $n \in \mathbb{Z}$ .

*Do yourself***Question # 3**Find the value of  $\theta$  satisfying the following equation:

$$3\tan^2\theta + 2\sqrt{3}\tan\theta + 1 = 0$$

**Solution**     $3\tan^2\theta + 2\sqrt{3}\tan\theta + 1 = 0$ 

$$\Rightarrow (\sqrt{3}\tan\theta)^2 + 2(\sqrt{3}\tan\theta)(1) + (1)^2 = 0$$

$$\Rightarrow (\sqrt{3}\tan\theta + 1)^2 = 0$$

$$\Rightarrow (\sqrt{3}\tan\theta + 1) = 0$$

$$\Rightarrow \sqrt{3}\tan\theta = -1$$

$$\Rightarrow \tan\theta = -\frac{1}{\sqrt{3}}$$

$$\Rightarrow \theta = \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$$

$$\Rightarrow \theta = \frac{5\pi}{6}$$

$\because$  period of  $\tan\theta$  is  $\pi$

$$\therefore \text{general value of } \theta = \frac{5\pi}{6} + n\pi, n \in \mathbb{Z}$$

**Question # 4**Find the value of  $\theta$  satisfying the following equation:

$$\tan^2\theta - \sec\theta - 1 = 0$$

**Solution**     $\tan^2\theta - \sec\theta - 1 = 0$ 

$$\Rightarrow (\sec^2\theta - 1) - \sec\theta - 1 = 0$$

$$\Rightarrow \sec^2\theta - 1 - \sec\theta - 1 = 0$$

$$\Rightarrow \sec^2\theta - \sec\theta - 2 = 0$$

$$\Rightarrow \sec\theta(\sec\theta - 2) + 1(\sec\theta - 2) = 0$$

$$\Rightarrow (\sec\theta + 1)(\sec\theta - 2) = 0$$

$$\Rightarrow (\sec\theta + 1) = 0 \quad \text{or} \quad (\sec\theta - 2) = 0$$

$$\Rightarrow \sec\theta = -1 \quad \text{or} \quad \sec\theta = +2$$

$$\Rightarrow \cos\theta = -1 \quad \text{or} \quad \cos\theta = \frac{1}{2}$$

$$\Rightarrow \theta = \cos^{-1}(-1) \quad \text{or} \quad \theta = \cos^{-1}\left(\frac{1}{2}\right)$$

$$\Rightarrow \theta = \frac{3\pi}{2} \quad \text{or} \quad \theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

$\because$  period of  $\cos \theta$  is  $2\pi$

$$\therefore \text{general value of } \theta = \frac{3\pi}{2} + 2n\pi, \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi \text{ where } n \in \mathbb{Z}$$

**Question # 5**

Find the value of  $\theta$  satisfying the following equation:

$$2\sin \theta + \cos^2 \theta - 1 = 0$$

**Solution**  $2\sin \theta + \cos^2 \theta - 1 = 0$

$$\Rightarrow 2\sin \theta + 1 - \sin^2 \theta - 1 = 0$$

$$\Rightarrow -\sin^2 \theta + 2\sin \theta = 0$$

$$\Rightarrow -\sin \theta (\sin \theta - 2) = 0$$

$$\Rightarrow -\sin \theta = 0 \quad \text{or} \quad \sin \theta - 2 = 0$$

$$\Rightarrow \sin \theta = 0 \quad \text{or} \quad \sin \theta = 2$$

$$\Rightarrow \theta = \sin^{-1}(0) \quad \text{Which does not hold as } \sin \theta \in [-1, 1]$$

$$\Rightarrow \theta = 0, \pi$$

$\because$  period of  $\sin \theta$  is  $2\pi$

$$\therefore \text{general value of } \theta = 0 + 2n\pi, \pi + 2n\pi \\ = 2n\pi, \pi + 2n\pi \quad \text{where } n \in \mathbb{Z}$$

**Question # 6**

Find the value of  $\theta$  satisfying the following equation:

$$2\sin^2 \theta - \sin \theta = 0$$

**Solution**  $2\sin^2 \theta - \sin \theta = 0$

$$\Rightarrow \sin \theta (2\sin \theta - 1) = 0 \Rightarrow \sin \theta = 0 \quad \text{or} \quad 2\sin \theta - 1 = 0$$

*Now do yourself*

**Question # 7**

Find the value of  $\theta$  satisfying the following equation:

$$3\cos^2 \theta - 2\sqrt{3} \sin \theta \cos \theta - 3\sin^2 \theta = 0$$

**Solution**  $3\cos^2 \theta - 2\sqrt{3} \sin \theta \cos \theta - 3\sin^2 \theta = 0$

Dividing throughout by  $\cos^2 \theta$

$$\frac{3\cos^2 \theta}{\cos^2 \theta} - \frac{2\sqrt{3} \sin \theta \cos \theta}{\cos^2 \theta} - \frac{3\sin^2 \theta}{\cos^2 \theta} = 0$$

$$\Rightarrow 3 - 2\sqrt{3} \tan \theta - 3\tan^2 \theta = 0$$

$$\Rightarrow -3\tan^2 \theta - 2\sqrt{3} \tan \theta + 3 = 0$$

$$\Rightarrow 3\tan^2 \theta + 2\sqrt{3} \tan \theta - 3 = 0$$

*×ing by -1*

$$\Rightarrow \tan \theta = \frac{-2\sqrt{3} \pm \sqrt{(2\sqrt{3})^2 - 4(3)(-3)}}{2(3)}$$

$$\begin{aligned}\Rightarrow \tan \theta &= \frac{-2\sqrt{3} \pm \sqrt{12+36}}{6} = \frac{-2\sqrt{3} \pm \sqrt{48}}{6} \\&= \frac{-2\sqrt{3} \pm \sqrt{16 \times 3}}{6} = \frac{-2\sqrt{3} \pm 4\sqrt{3}}{6} \\ \Rightarrow \tan \theta &= \frac{-2\sqrt{3} + 4\sqrt{3}}{6} = \frac{2\sqrt{3}}{6} \quad \text{or} \quad \tan \theta = \frac{-2\sqrt{3} - 4\sqrt{3}}{6} = \frac{-6\sqrt{3}}{6} \\ \Rightarrow \tan \theta &= \frac{\sqrt{3}}{3} = \frac{\sqrt{3}}{(\sqrt{3})^2} = \frac{1}{\sqrt{3}} \quad \text{or} \quad \Rightarrow \tan \theta = -\sqrt{3} \\ \Rightarrow \theta &= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) \quad \text{or} \quad \theta = \tan^{-1}(-\sqrt{3}) \\ \Rightarrow \theta &= \frac{\pi}{6} \quad \text{or} \quad \theta = \frac{11\pi}{6} \\ \because \text{period of } \tan \theta &\text{ is } \pi \\ \therefore \text{general value of } \theta &= \frac{\pi}{6} + n\pi, \frac{11\pi}{6} + n\pi \quad \text{where } n \in \mathbb{Z}.\end{aligned}$$

**Question # 8**Find the value of  $\theta$  satisfying the following equation:

$$4\sin^2 \theta - 8\cos \theta + 1 = 0$$

**Solution**

$$\begin{aligned}4\sin^2 \theta - 8\cos \theta + 1 &= 0 \\ \Rightarrow 4(1 - \cos^2 \theta) - 8\cos \theta + 1 &= 0 \\ \Rightarrow 4 - 4\cos^2 \theta - 8\cos \theta + 1 &= 0 \\ \Rightarrow -4\cos^2 \theta - 8\cos \theta + 5 &= 0 \\ \Rightarrow 4\cos^2 \theta + 8\cos \theta - 5 &= 0 \\ \Rightarrow 4\cos^2 \theta + 10\cos \theta - 2\cos \theta - 5 &= 0 \\ \Rightarrow 2\cos \theta(2\cos \theta + 5) - 1(2\cos \theta + 5) &= 0 \\ \Rightarrow (2\cos \theta + 5)(2\cos \theta - 1) &= 0 \\ \Rightarrow 2\cos \theta + 5 &= 0 \quad \text{or} \quad 2\cos \theta - 1 = 0 \\ \Rightarrow 2\cos \theta &= -5 \quad \text{or} \quad 2\cos \theta = 1 \\ \Rightarrow \cos \theta &= \frac{-5}{2} \quad \text{or} \quad \cos \theta = \frac{1}{2} \\ \Rightarrow \theta &= \cos^{-1}\left(\frac{-5}{2}\right) \quad \text{or} \quad \theta = \cos^{-1}\left(\frac{1}{2}\right)\end{aligned}$$

Which is not possible as  $\cos \theta \in [-1, 1]$       or       $\theta = \frac{\pi}{3}, \frac{5\pi}{3}$  $\because$  period of  $\cos \theta$  is  $2\pi$ 

$$\therefore \text{general value of } \theta = \frac{\pi}{3} + 2\pi n, \frac{5\pi}{3} + 2n\pi \quad \text{where } n \in \mathbb{Z}.$$

**Question # 9**

Find the solution set;  $\sqrt{3} \tan x - \sec x - 1 = 0$

**Solution**  $\sqrt{3} \tan x - \sec x - 1 = 0 \dots\dots\text{(i)}$

$$\Rightarrow \sqrt{3} \frac{\sin x}{\cos x} - \frac{1}{\cos x} - 1 = 0$$

$$\Rightarrow \sqrt{3} \sin x - 1 - \cos x = 0 \quad \times \text{ing by } \cos \theta.$$

$$\Rightarrow \sqrt{3} \sin x - 1 = \cos x$$

On squaring both sides,

$$(\sqrt{3} \sin x - 1)^2 = (\cos x)^2$$

$$\Rightarrow 3 \sin^2 x - 2\sqrt{3} \sin x + 1 = \cos^2 x$$

$$\Rightarrow 3 \sin^2 x - 2\sqrt{3} \sin x + 1 = 1 - \sin^2 x$$

$$\Rightarrow 3 \sin^2 x - 2\sqrt{3} \sin x + 1 - 1 + \sin^2 x = 0$$

$$\Rightarrow 4 \sin^2 x - 2\sqrt{3} \sin x = 0$$

$$\Rightarrow 2 \sin x (2 \sin x - \sqrt{3}) = 0$$

$$\Rightarrow 2 \sin x = 0$$

or

$$2 \sin x = \sqrt{3}$$

$$\Rightarrow \sin x = 0$$

or

$$\sin x = \frac{\sqrt{3}}{2}$$

$$\Rightarrow x = \sin^{-1}(0)$$

or

$$x = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow x = 0, \pi$$

or

$$x = \frac{\pi}{3}, \frac{2\pi}{3}$$

Now to check extraneous roots put  $x = 0$  in (i)

$$\text{L.H.S} = \sqrt{3} \tan(0) - \sec(0) - 1 = 0 - 1 - 1 = -2 \neq 0 = \text{R.H.S}$$

Implies that  $x = 0$  is an extraneous root of given equation.

Now put  $x = \pi$  in (i)

$$\text{L.H.S} = \sqrt{3} \tan(\pi) - \sec(\pi) - 1 = 0 - (-1) - 1 = 0 = \text{R.H.S}$$

Implies that  $x = \pi$  is a root of the equation.

Now put  $x = \frac{\pi}{3}$  in (i)

$$\text{L.H.S} = \sqrt{3} \tan\left(\frac{\pi}{3}\right) - \sec\left(\frac{\pi}{3}\right) - 1 = \sqrt{3}(\sqrt{3}) - 2 - 1 = 1 - 2 - 1 = 0 = \text{R.H.S}$$

Implies that  $x = \frac{\pi}{3}$  is a root of given equation.. Since period of tan is  $\pi$ .

Now put  $x = \frac{2\pi}{3}$  in (i)

$$\begin{aligned} \text{L.H.S} &= \sqrt{3} \tan\left(\frac{2\pi}{3}\right) - \sec\left(\frac{2\pi}{3}\right) - 1 \\ &= \sqrt{3}(-\sqrt{3}) - (-2) - 1 = -3 + 2 - 1 = -2 = \text{R.H.S} \end{aligned}$$

Implies  $x = \frac{2\pi}{3}$  is an extraneous root of given equation.

$\therefore$  period of  $\sin x$  is  $2\pi$

$\therefore$  general values of  $x = \pi + 2n\pi, \frac{\pi}{3} + 2n\pi$

$$\text{Solution Set} = \{\pi + 2n\pi\} \cup \left\{ \frac{\pi}{3} + 2n\pi \right\} \quad \text{where } n \in \mathbb{Z}.$$

### Question # 10

Find the solution set;  $\cos 2x = \sin 3x$

**Solution**  $\cos 2x = \sin 3x$

$$\begin{aligned} &\Rightarrow \cos^2 x - \sin^2 x = 3\sin x - 4\sin^3 x && \because \cos 2x = \cos^2 x - \sin^2 x \\ &\Rightarrow \cos^2 x - \sin^2 x - 3\sin x + 4\sin^3 x = 0 && \sin 3x = 3\sin x - 4\sin^3 x \\ &\Rightarrow (1 - \sin^2 x) - \sin^2 x - 3\sin x + 4\sin^3 x = 0 \\ &\Rightarrow 1 - 2\sin^2 x - 3\sin x + 4\sin^3 x = 0 \\ &\Rightarrow 4\sin^3 x - 2\sin^2 x - 3\sin x + 1 = 0 \end{aligned}$$

Take  $\sin x = 1$  as a root then by synthetic division

$$\begin{array}{c|cccc} 1 & 4 & -2 & -3 & 1 \\ \downarrow & & 4 & 2 & -1 \\ \hline & 4 & 2 & -1 & 0 \end{array}$$

$$\Rightarrow (\sin x - 1)(4\sin^2 x + 2\sin x - 1) = 0$$

$$\Rightarrow \sin x - 1 = 0 \quad \text{or} \quad 4\sin^2 x + 2\sin x - 1 = 0$$

$$\Rightarrow \sin x = 1 \quad \text{or} \quad \sin x = \frac{-2 \pm \sqrt{(2)^2 - 4(4)(-1)}}{2(4)}$$

$$\Rightarrow x = \sin^{-1}(1) \quad \text{or} \quad \sin x = \frac{-2 \pm \sqrt{4+16}}{8} = \frac{-2 \pm \sqrt{20}}{8}$$

$$\Rightarrow x = \frac{\pi}{2} \quad \text{or} \quad \sin x = \frac{-2 + \sqrt{20}}{8} \quad \text{or} \quad \sin x = \frac{-2 - \sqrt{20}}{8}$$

$$\sin x = 0.309 \quad \text{or} \quad \sin x = -0.809$$

$$\Rightarrow x = \sin^{-1}(0.309) \quad \text{or} \quad x = \sin^{-1}(-0.809)$$

$$\approx 18, 162 \quad \text{or} \quad \approx 234, 306$$

$$\Rightarrow x = 18 \cdot \frac{\pi}{180}, 162 \cdot \frac{\pi}{180} \quad \text{or} \quad x = 234 \cdot \frac{\pi}{180}, 306 \cdot \frac{\pi}{180}$$

$$= \frac{\pi}{10}, \frac{9\pi}{10} \quad \text{or} \quad = \frac{13\pi}{10}, \frac{17\pi}{10}$$

$\therefore$  period of  $\sin x$  is  $2\pi$

$$\therefore \text{ general value of } x = \frac{\pi}{10} + 2n\pi, \frac{9\pi}{10} + 2n\pi, \frac{13\pi}{10} + 2n\pi, \frac{17\pi}{10} + 2n\pi, \frac{\pi}{2} + 2n\pi$$

$$\text{S. Set} = \left\{ \frac{\pi}{10} + 2n\pi \right\} \cup \left\{ \frac{9\pi}{10} + 2n\pi \right\} \cup \left\{ \frac{13\pi}{10} + 2n\pi \right\} \cup \left\{ \frac{17\pi}{10} + 2n\pi \right\} \cup \left\{ \frac{\pi}{2} + 2n\pi \right\}$$


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**Question # 11**Find the solution set;  $\sec 3\theta = \sec \theta$ **Solution**

$$\begin{aligned} \sec 3\theta &= \sec \theta \\ \Rightarrow \frac{1}{\cos 3\theta} &= \frac{1}{\cos \theta} \\ \Rightarrow \cos 3\theta &= \cos \theta \\ \Rightarrow 4\cos^3 \theta - 3\cos \theta &= \cos \theta \quad \because \cos 3\theta = 4\cos^3 \theta - 3\cos \theta \\ \Rightarrow 4\cos^3 \theta - 3\cos \theta - \cos \theta &= 0 \\ \Rightarrow 4\cos^3 \theta - 4\cos \theta &= 0 \\ \Rightarrow 4\cos \theta (\cos^2 \theta - 1) &= 0 \\ \Rightarrow 4\cos \theta = 0 \text{ or } \cos^2 \theta - 1 &= 0 \\ \Rightarrow \cos \theta = 0 \text{ or } \cos^2 \theta &= 1 \\ \Rightarrow \theta = \cos^{-1}(0) &\text{ or } \cos \theta = \pm 1 \\ \Rightarrow \theta = \cos^{-1}(1), \theta &= \cos^{-1}(-1) \\ \Rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2} &\text{ or } \theta = 0, \pi \\ \because \text{ period of } \cos \theta \text{ is } 2\pi & \end{aligned}$$

$$\therefore \text{ general values of } \theta = \frac{\pi}{2} + 2n\pi, \frac{3\pi}{2} + 2n\pi, 0 + 2n\pi, \pi + 2n\pi$$

$$= \frac{\pi}{2} + 2n\pi, \frac{3\pi}{2} + 2n\pi, n\pi$$

$$\text{S. Set} = \left\{ \frac{\pi}{2} + 2n\pi \right\} \cup \left\{ \frac{3\pi}{2} + 2n\pi \right\} \cup \{n\pi\} \quad \text{where } n \in \mathbb{Z}.$$


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**Question # 12**Find the solution set;  $\tan 2\theta + \cot \theta = 0$ **Solution**

$$\begin{aligned} \tan 2\theta + \cot \theta &= 0 \\ \Rightarrow \tan 2\theta &= -\cot \theta \\ \Rightarrow \frac{\sin 2\theta}{\cos 2\theta} &= -\frac{\cos \theta}{\sin \theta} \\ \Rightarrow \frac{2\sin \theta \cos \theta}{\cos^2 \theta - \sin^2 \theta} &= -\frac{\cos \theta}{\sin \theta} \\ \Rightarrow (2\sin \theta \cos \theta)(\sin \theta) &= (-\cos \theta)(\cos^2 \theta - \sin^2 \theta) \\ \Rightarrow 2\sin^2 \theta \cos \theta &= -\cos^3 \theta + \sin^2 \theta \cos \theta \\ \Rightarrow 2\sin^2 \theta \cos \theta + \cos^3 \theta - \sin^2 \theta \cos \theta &= 0 \\ \Rightarrow \sin^2 \theta \cos \theta + \cos^3 \theta &= 0 \end{aligned}$$

$$\begin{aligned}\Rightarrow \cos \theta (\sin^2 \theta + \cos^2 \theta) &= 0 \quad \Rightarrow \cos \theta (1) = 0 \\ \Rightarrow \theta &= \cos^{-1}(0) \\ &= \frac{\pi}{2}, \frac{3\pi}{2} \\ \therefore \text{ period of } \cos \theta \text{ is } 2\pi \quad &\therefore \text{ general values of } \theta = \frac{\pi}{2} + 2n\pi, \frac{3\pi}{2} + 2n\pi \\ \therefore \text{ S. Set} &= \left\{ \frac{\pi}{2} + 2n\pi \right\} \cup \left\{ \frac{3\pi}{2} + 2n\pi \right\} \quad \text{where } n \in \mathbb{Z}. \end{aligned}$$

**Question # 13**Find the solution set;  $\sin 2x + \sin x = 0$ 

$$\begin{aligned}\text{Solution} \quad \sin 2x + \sin x &= 0 \\ \Rightarrow 2\sin x \cos x + \sin x &= 0 \\ \Rightarrow \sin x(2\cos x + 1) &= 0 \\ \Rightarrow \sin x = 0 &\quad \text{or} \quad 2\cos x + 1 = 0 \\ \Rightarrow x = \sin^{-1}(0) &\quad \text{or} \quad \cos x = -\frac{1}{2} \\ \Rightarrow x = \cos^{-1}\left(-\frac{1}{2}\right) &\\ \Rightarrow x = 0, \pi &\quad \text{or} \quad x = \frac{2\pi}{3}, \frac{4\pi}{3} \\ \therefore \text{ period of } \sin x \text{ and } \cos x \text{ is } 2\pi \quad &\therefore \sin 2\theta = 2\sin \theta \cos \theta \end{aligned}$$

$$\begin{aligned}\therefore \text{ general values of } x &= 0 + 2n\pi, \pi + 2n\pi, \frac{2\pi}{3} + 2n\pi, \frac{4\pi}{3} + 2n\pi \\ &= n\pi, \frac{2\pi}{3} + 2n\pi, \frac{4\pi}{3} + 2n\pi \end{aligned}$$

$$\text{So solution set} = \left\{ n\pi \right\} \cup \left\{ \frac{2\pi}{3} + 2n\pi \right\} \cup \left\{ \frac{4\pi}{3} + 2n\pi \right\} \quad \text{where } n \in \mathbb{Z}.$$

**Question # 14**Find the solution set;  $\sin 4x - \sin 2x = \cos 3x$ 

$$\begin{aligned}\text{Solution} \quad \sin 4x - \sin 2x &= \cos 3x \\ \Rightarrow 2\cos\left(\frac{4x+2x}{2}\right)\sin\left(\frac{4x-2x}{2}\right) &= \cos 3x \\ \Rightarrow 2\cos 3x \sin x - \cos 3x &= 0 \\ \Rightarrow \cos 3x(2\sin x - 1) &= 0 \\ \Rightarrow \cos 3x = 0 &\quad \text{or} \quad 2\sin x - 1 = 0 \\ \Rightarrow 3x = \cos^{-1}(0) &\quad , \quad \sin x = \frac{1}{2} \\ \Rightarrow 3x = \frac{\pi}{2}, \frac{3\pi}{2} &\quad , \quad x = \sin^{-1}\left(\frac{1}{2}\right) \end{aligned}$$

$$\Rightarrow x = \frac{\pi}{6}, \frac{\pi}{2}, \dots, x = \frac{\pi}{6}, \frac{5\pi}{6}$$

Since period of  $\cos 3x$  is  $\frac{2\pi}{3}$  and period of  $\sin x$  is  $2\pi$

$$\therefore \text{general values of } x = \frac{\pi}{6} + \frac{2n\pi}{3}, \frac{\pi}{2} + \frac{2n\pi}{3}, \frac{\pi}{6} + 2n\pi, \frac{5\pi}{6} + 2n\pi$$

$$\text{So solution set} = \left\{ \frac{\pi}{6} + \frac{2n\pi}{3} \right\} \cup \left\{ \frac{\pi}{2} + \frac{2n\pi}{3} \right\} \cup \left\{ \frac{\pi}{6} + 2n\pi \right\} \cup \left\{ \frac{5\pi}{6} + 2n\pi \right\} \text{ where } n \in \mathbb{Z}.$$

### Question # 15

Find the solution set;  $\sin x + \cos 3x = \cos 5x$

**Solution**

$$\sin x + \cos 3x = \cos 5x$$

$$\Rightarrow \sin x = \cos 5x - \cos 3x$$

$$\Rightarrow \sin x = -2 \sin\left(\frac{5x+3x}{2}\right) \sin\left(\frac{5x-3x}{2}\right)$$

$$\Rightarrow \sin x = -2 \sin 4x \sin x$$

$$\Rightarrow \sin x + 2 \sin 4x \sin x = 0$$

$$\Rightarrow \sin x(1 + 2 \sin 4x) = 0$$

$$\Rightarrow \sin x = 0 \quad \text{or} \quad 1 + 2 \sin 4x = 0$$

$$\Rightarrow x = \sin^{-1}(0) \quad \text{or} \quad \sin 4x = -\frac{1}{2} \Rightarrow 4x = \sin^{-1}\left(-\frac{1}{2}\right)$$

$$\Rightarrow x = 0, \pi \quad \text{or} \quad 4x = \frac{7\pi}{6}, \frac{11\pi}{6} \Rightarrow x = \frac{7\pi}{24}, \frac{11\pi}{24}$$

Since period of  $\sin x$  is  $2\pi$  and period of  $\sin 4x$  is  $\frac{2\pi}{4} = \frac{\pi}{2}$

$$\therefore \text{general values of } x = 0 + 2n\pi, \pi + 2n\pi, \frac{7\pi}{24} + \frac{n\pi}{2}, \frac{11\pi}{24} + \frac{n\pi}{2}$$

$$\text{So solution set} = \{2n\pi\} \cup \{\pi + 2n\pi\} \cup \left\{ \frac{7\pi}{24} + \frac{n\pi}{2} \right\} \cup \left\{ \frac{11\pi}{24} + \frac{n\pi}{2} \right\} \text{ where } n \in \mathbb{Z}.$$

### Question # 16

Find the solution set;  $\sin 3x + \sin 2x + \sin x = 0$

**Solution**

$$\sin 3x + \sin 2x + \sin x = 0$$

$$\Rightarrow (\sin 3x + \sin x) + \sin 2x = 0$$

$$\Rightarrow 2 \sin\left(\frac{3x+x}{2}\right) \cos\left(\frac{3x-x}{2}\right) + \sin 2x = 0$$

$$\Rightarrow 2 \sin 2x \cos x + \sin 2x = 0$$

$$\Rightarrow \sin 2x(2 \cos x + 1) = 0$$

$$\Rightarrow \sin 2x = 0 \quad \text{or} \quad 2 \cos x + 1 = 0$$

$$\Rightarrow 2x = \sin^{-1}(0) \quad \text{or} \quad \cos x = -\frac{1}{2}$$

$$\Rightarrow 2x = 0, \pi \quad \text{or} \quad x = \cos^{-1}\left(-\frac{1}{2}\right)$$

$$\Rightarrow x = 0, \frac{\pi}{2} \quad \text{or} \quad x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

Since period of  $\sin 2x$  is  $\frac{2\pi}{2} = \pi$  and period of  $\cos x$  is  $2\pi$

$$\therefore \text{general values of } x = 0 + n\pi, \frac{\pi}{2} + n\pi, \frac{2\pi}{3} + 2n\pi, \frac{4\pi}{3} + 2n\pi$$

$$\text{S. Set} = \{n\pi\} \cup \left\{\frac{\pi}{2} + n\pi\right\} \cup \left\{\frac{2\pi}{3} + 2n\pi\right\} \cup \left\{\frac{4\pi}{3} + 2n\pi\right\} \quad \text{where } n \in \mathbb{Z}.$$

**Question # 17**

Find the solution set;  $\sin 7x - \sin x = \sin 3x$

**Solution**

$$\sin 7x - \sin x = \sin 3x$$

$$\Rightarrow 2\cos\left(\frac{7x+x}{2}\right)\sin\left(\frac{7x-x}{2}\right) = \sin 3x$$

$$\Rightarrow 2\cos 4x \sin 3x - \sin 3x = 0$$

$$\Rightarrow \sin 3x(2\cos 4x - 1) = 0$$

$$\Rightarrow \sin 3x = 0 \quad \text{or} \quad 2\cos 4x - 1 = 0$$

$$\Rightarrow 3x = \sin^{-1}(0) \quad \text{or} \quad \cos 4x = \frac{1}{2}$$

$$\Rightarrow 3x = 0, \pi \quad \text{or} \quad 4x = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$\Rightarrow x = 0, \frac{\pi}{3} \quad \text{or} \quad x = \frac{\pi}{12}, \frac{5\pi}{12}$$

Since period of  $\sin 3x$  is  $\frac{2\pi}{3}$  and period of  $\cos 4x$  is  $\frac{2\pi}{4} = \frac{\pi}{2}$

$$\therefore \text{general values of } x = 0 + \frac{2n\pi}{3}, \frac{\pi}{3} + \frac{2n\pi}{3}, \frac{\pi}{12} + \frac{n\pi}{2}, \frac{5\pi}{12} + \frac{n\pi}{2}$$

$$\text{So S. set} = \left\{\frac{2n\pi}{3}\right\} \cup \left\{\frac{\pi}{3} + \frac{2n\pi}{3}\right\} \cup \left\{\frac{\pi}{12} + \frac{n\pi}{2}\right\} \cup \left\{\frac{5\pi}{12} + \frac{n\pi}{2}\right\} \quad \text{where } n \in \mathbb{Z}.$$

**Question # 18**

Find the solution set;  $\sin x + \sin 3x + \sin 5x = 0$

**Solution**

$$\sin x + \sin 3x + \sin 5x = 0$$

$$\Rightarrow (\sin 5x + \sin x) + \sin 3x = 0$$

$$\Rightarrow 2\sin\left(\frac{5x+x}{2}\right) \cdot \cos\left(\frac{5x-x}{2}\right) + \sin 3x = 0$$

$$\Rightarrow 2\sin 3x \cos 2x + \sin 3x = 0$$

$$\Rightarrow \sin 3x(2\cos 2x + 1) = 0$$

$$\Rightarrow \sin 3x = 0 \quad \text{or} \quad 2\cos 2x + 1 = 0$$

$$\Rightarrow 3x = \sin^{-1}(0) \quad \text{or} \quad 2\cos 2x = -1 \Rightarrow 2x = \cos^{-1}\left(-\frac{1}{2}\right)$$

$$\Rightarrow 3x = 0, \pi \quad \text{or} \quad 2x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

$$\Rightarrow x = 0, \frac{\pi}{3} \quad \text{or} \quad x = \frac{\pi}{3}, \frac{2\pi}{3}$$

Since period of  $\sin 3x$  is  $\frac{2\pi}{3}$  and period of  $\cos 2x$  is  $\frac{2\pi}{2} = \pi$

$\therefore$  general values of  $x = 0 + \frac{2n\pi}{3}, \frac{\pi}{3} + \frac{2n\pi}{3}, \frac{\pi}{3} + n\pi, \frac{2\pi}{3} + n\pi$

$$S.\text{Set} = \left\{ \frac{2n\pi}{3} \right\} \cup \left\{ \frac{\pi}{3} + \frac{2n\pi}{3} \right\} \cup \left\{ \frac{\pi}{3} + n\pi \right\} \cup \left\{ \frac{2\pi}{3} + n\pi \right\} \text{ where } n \in \mathbb{Z}$$

### Question # 19

Find the solution set;  $\sin \theta + \sin 3\theta + \sin 5\theta + \sin 7\theta = 0$

$$\text{Solution} \quad \sin \theta + \sin 3\theta + \sin 5\theta + \sin 7\theta = 0$$

$$\Rightarrow (\sin 7\theta + \sin \theta) + (\sin 5\theta + \sin 3\theta) = 0$$

$$\Rightarrow 2\sin\left(\frac{7\theta+\theta}{2}\right)\cos\left(\frac{7\theta-\theta}{2}\right) + 2\sin\left(\frac{5\theta+3\theta}{2}\right)\cos\left(\frac{5\theta-3\theta}{2}\right) = 0$$

$$\Rightarrow 2\sin 4\theta \cos 3\theta + 2\sin 4\theta \cos \theta = 0$$

$$\Rightarrow 2\sin 4\theta (\cos 3\theta + \cos \theta) = 0$$

$$\Rightarrow 2\sin 4\theta \left( 2\cos\left(\frac{3\theta+\theta}{2}\right)\cos\left(\frac{3\theta-\theta}{2}\right) \right) = 0$$

$$\Rightarrow 4\sin 4\theta \cos 2\theta \cos \theta = 0$$

$$\Rightarrow \sin 4\theta = 0 \quad \text{or} \quad \cos 2\theta = 0 \quad \text{or} \quad \cos \theta = 0$$

$$\Rightarrow 4\theta = \sin^{-1}(0), \quad 2\theta = \cos^{-1}(0), \quad \theta = \cos^{-1}(0)$$

$$\Rightarrow 4\theta = 0, \pi \quad \text{or} \quad 2\theta = \frac{\pi}{2}, \frac{3\pi}{2} \quad \text{or} \quad \theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\Rightarrow \theta = 0, \frac{\pi}{4} \quad \text{or} \quad \theta = \frac{\pi}{4}, \frac{3\pi}{4}$$

Since period of  $\sin 4\theta$  is  $\frac{2\pi}{4} = \frac{\pi}{2}$ ,  $\cos 2\theta$  is  $\frac{2\pi}{2} = \pi$  and  $\cos \theta$  is  $2\pi$

$\therefore$  general values of  $\theta = 0 + \frac{n\pi}{2}, \frac{\pi}{4} + \frac{n\pi}{2}, \frac{\pi}{4} + n\pi, \frac{3\pi}{4} + n\pi, \frac{\pi}{2} + 2n\pi, \frac{3\pi}{2} + 2n\pi$

$$S.\text{Set} = \left\{ \frac{n\pi}{2} \right\} \cup \left\{ \frac{\pi}{4} + \frac{n\pi}{2} \right\} \cup \left\{ \frac{\pi}{4} + n\pi \right\} \cup \left\{ \frac{3\pi}{4} + n\pi \right\} \cup \left\{ \frac{\pi}{2} + 2n\pi \right\} \cup \left\{ \frac{3\pi}{2} + 2n\pi \right\}.$$

### Question # 20

Find the solution set;  $\cos \theta + \cos 3\theta + \cos 5\theta + \cos 7\theta = 0$

$$\text{Solution} \quad \cos \theta + \cos 3\theta + \cos 5\theta + \cos 7\theta = 0$$

$$\Rightarrow (\cos 7\theta + \cos \theta) + (\cos 5\theta + \cos 3\theta) = 0$$

$$\begin{aligned}
 &\Rightarrow 2\cos\left(\frac{7\theta+\theta}{2}\right)\cos\left(\frac{7\theta-\theta}{2}\right) + 2\cos\left(\frac{5\theta+3\theta}{2}\right)\cos\left(\frac{5\theta-3\theta}{2}\right) = 0 \\
 &\Rightarrow 2\cos 4\theta \cos 3\theta + 2\cos 4\theta \cos \theta = 0 \\
 &\Rightarrow 2\cos 4\theta (\cos 3\theta + \cos \theta) = 0 \\
 &\Rightarrow 2\cos 4\theta \left(2\cos\left(\frac{3\theta+\theta}{2}\right)\cos\left(\frac{3\theta-\theta}{2}\right)\right) = 0 \\
 &\Rightarrow 4\cos 4\theta \cos 2\theta \cos \theta = 0
 \end{aligned}$$

*Now do yourself as above question.*

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**Govt. Ghazali Degree College, Jhang**(Important Short Questions)  
Course: Algebra and Trigonometry

Chapter # 14

*Solutions of Trigonometric Equations*

Following short questions are selected from previous 5 years papers of different boards. Solve these at your own to perform well in annual exams.

1. Solve  $\sec^2 \theta = \frac{4}{3}$  in  $[0, 2\pi]$ .
2. Solve  $\sin x + \cos x = 0$ , where  $x \in [0, 2\pi]$ .
3. Solve  $2\sin^2 \theta - \sin \theta = 0$ , where  $\theta \in [0, 2\pi]$ .
4. Find the values of  $\theta \in [0, \pi]$ , if  $4\sin^2 \theta - 8\cos \theta + 1 = 0$ .
5. Find the solution of  $\cos x - 1 = 0$  in  $[0, 2\pi]$ .
6. Solve the equation  $\sin x = \frac{1}{2}$ , where  $x \in [0, 2\pi]$ .
7. Solve the equation  $1 + \cos x = 0$ .
8. Solve the trigonometric equation  $\tan \theta = \frac{1}{\sqrt{3}}$ .
9. Find the solution of  $\sec x = -2$  which lie in  $[0, 2\pi]$ .
10. Find the solution of  $\sin x = -\frac{\sqrt{3}}{2}$  which lie in  $[0, 2\pi]$ .
11. Find the solution set of the equation  $\sin x = \frac{1}{2}$ .

*Best of Luck*

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